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Project Report	ETS-47
	J. M. Sorvari
Accuracy Requirements for GEODSS Photometry	
	26 July 1979

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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ACCURACY REQUIREMENTS FOR GEODSS PHOTOMETRY

J. M. SORVARI

Group 94

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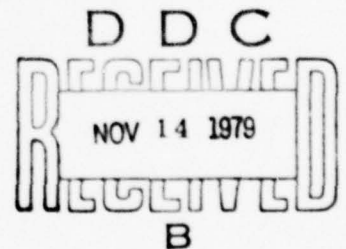
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ABSTRACT

Requirements on the accuracy of GEODSS photometry have been derived based on the stated goals of GEODSS photometry. Fulfilling these requirements will ensure that the data are of sufficient quality for their intended purpose. Calculations based on the properties of the proposed system show that the requirements can be met if care is taken in the detailed design and data processing techniques.

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CONTENTS

I.   INTRODUCTION

The GEODSS System Performance Specifications (reference 8; hereafter "the Specification") includes a statement of the goals of GEODSS photometry and several performance characteristics intended to "guarantee" achievement of the stated goals. Appendix A quotes several paragraphs of the Specification which apply to photometry. Unfortunately, there are two defects in the Specification as it stands. First, the definitions of some parameters are not complete. In particular the "accuracy" requirement is not related to such well defined parameters as standard error. In other sections of the specification, accuracy is explicitly defined as 3σ precision of measured quantities. Second, the connection between the performance characteristics and the system goals is not clear. This report specifies the required accuracy in a well defined way and derives the relevant performance characteristics directly from the system goals. This is done in such a way that the numbers derived can be viewed as explications of the existing specifications. None of the parameters derived here is more stringent than a reasonable interpretation of the existing specifications.

Paragraph 3.1.2.4 of the Specification lists the four goals of GEODSS photometry. These are

1) Determination of Object Class

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2) Determination of Motion Class (*i.e.*, stable, unstable, spin stabilized)

3) Determination of Spin/Tumble Period

4) Determination of Maximum/Average Magnitude

These goals are to be attained by means of measurements of brightness as a function of time. Although the technique is not specified, the only practical way of obtaining any reasonable accuracy is by making differential measurements with respect to a catalog of reference standards. Sections II - V of this report deal, respectively, with the reference catalog, error, detection of spin, and measurement of the phase function. In section VI, the results of sections II - V are used to derive the performance characteristics needed for achievement of the four goals listed. In order to insure relevancy of the numbers used, a photometric reference object has been defined. This is in parallel with the reference object used in discussing performance of the imaging sensors of the GEODSS system. Details of the reference and the measurement parameters are contained in appendix B.

It is important to recognize that any measurements are made in the natural system of the equipment. Although data in another system may be desired, the procedure followed must be: stabilize the natural system; correct to the GEODSS standard; attempt to transform to another standard system. The natural system so far considered is that one defined by the response of a GaAs

photocathode and atomospheric transmission (with slight modi-
fication due to mirror reflectivity and glass transmission). For
reasons made clear in section III as well as to improve stability,
it is useful to consider an alternative natural system defined as
the one above with the addition of a filter such as a Schott
GG435 filter. This filter is roughly a square cut-on filter with
the opaque/transparent transition at $\lambda = .435\mu$, blinding the
system to photons in the blue region of the spectrum, where
atmospheric transmission varies rapidly. Introduction of this
filter would reduce throughput to the photometer for two reasons.
Elimination of the extreme blue end of the spectrum results in a
loss of $\leq$ 9% of the photons in sunlight. Reflection at the two
air-glass interfaces causes a loss of an additional 9% of the
photons. Although this reflection loss can be reduced or even
eliminated through proper design the entire loss has been used in
all the calculations of performance with the GG435 filter. This
means that the results obtained here are pessimistic in this
regard, and that the actual performance can be expected to be
slightly better.

II.   THE CATALOG OF REFERENCE STARS

The method proposed for correction of GEODSS photometry for atmospheric extinction is essentially differential photometry similar to the method described by Hardie (2,§2.7). Applications of this method at the ETS are described by Sorvari and Beane (3) and by Sorvari (4). This method requires the existence of a catalog of approximately 6000 stars distributed more or less uniformly over the sky. This number and the uniformity derive from the requirement that a star always be available within about one-tenth air mass of any observed satellite*. The stars should all have approximately solar spectral characteristics so that measurements of the stars, at least, will not involve errors due to color effects. They should also have magnitudes approximately in the range 9 - 11 so that negligible shot noise and linear response are guaranteed. The magnitudes must be those in the natural system of the GEODSS equipment, which is taken to be the system established by the unfiltered GaAs response.

Production of such a catalog by actual observation at a GEODSS facility would take in excess of a dedicated year.

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*In order to be within 1/10 air mass of a satellite, the comparison star must be nearly the same elevation. For example, for a satellite at 1.5 air masses, about 42° elevation, the star must not differ in elevation by more than 3.5°. At two airmasses ($\sim 30^\circ$) the maximum permissible elevation difference decreases to 1.7°, and at three air masses ($\sim 20^\circ$) it decreases to 0.7°.

This is obviously not a satisfactory way to proceed. Instead advantage can be taken of the photometry done by the astronomical community by using the following procedure.

1) Establish a set of primary standard stars - once for GEODSS.

2) Compile a catalog of translated magnitudes - once for GEODSS.

3) Tie into the standard system - once per instrument.

Each of these steps requires a great deal of care in order to avoid inclusion of a large catalog error. The three steps are explained in more detail below.

The first step, the establishment of the set of primary standards, is by far the most critical step. It requires the accumulation of a fairly large body of observations obtained on high quality nights spread over a period of eight to twelve months. It is important that the instrumental system be as stable as possible over this period. The data reduction scheme is extremely important, since the establishment of the system magnitudes involves an extrapolation so that an inappropriate extinction model can lead to large errors in the catalog magnitudes. Because of the extreme width of the GaAs response, the "standard" astronomical models of extinction cannot be expected

to provide an adequate description of extinction. Astronomical filters are typically about 800Å wide or even narrower. Over this range in wavelength both the source spectral distribution and the atmospheric transmission can be reasonably well approximated as linear functions of wavelength. The unfiltered* GaAs response is nearly 6000Å wide. Over this range much higher order approximations to the atmospheric transmission and spectral distribution are needed. An appropriate model for GEODSS is presently under investigation. Fortunately this step need be done only once.

An important by-product of the establishment of the system standards is the development of a relationship between the GEODSS system and the UBV system (reference 9) or some other standard system. This makes the second step possible: the translation of magnitudes measured by the astronomical community onto the GEODSS system. Thus a reference catalog may be compiled without further observations. The catalog should conform to the description at the beginning of this section. If gaps are encountered, it will be necessary to fill them by direct observation. Several compendia of photometry exist (5,6). After a preliminary selection is made from one of these, it would probably be advisable to check the original references to clear out typographical

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*The short wavelength cutoff is provided by glass elements in the telescope.

errors and indirect references. Magnitudes based on a single
measurement should be avoided.

Although the equipment at all sites will be identical in a
commercial sense, the natural photometric systems will in fact
differ slightly. Because of this, the third step will be
necessary: translation of the standard catalog onto each local
natural system. A single night of photometric quality dedicated
to observing the system primary standards should be sufficient
for this. The ease of this tie-in depends upon relatively close
matching of natural systems. It is, therefore, important that
all decisions as to photomultiplier type, mirror coating, filters,
and other non-white optical components be reached before the
establishment of the standard system begins.

If the first and third steps are done carefully, they
should introduce only negligible error into GEODSS photometry.
Step number two is likely to be a significant error source.
There are two problems here. First the catalog, being derived
from many sources, will lack homogeneity. Second, an attempt to
translate between systems of such enormously different properties
as standard astronomical photometry and GEODSS photometry is
bound to be uncertain. For the present we shall use $0.^{m}04$ as
the catalog error. This is almost certain to be very optimistic.

The actual error will not be known until the catalog is compiled and an experimental series of observations is run. A value as high as $0\overset{m}{.}1$ is not impossible and so techniques must be found for dealing with this problem.

There are basically two approaches to dealing with a large value of error in the catalog. The first is to observe all the stars, as discussed above; this is obviously impracticable. The second is to use several stars. If three stars are used for each extinction correction, the standard error of the mean will be reduced by a factor of $\sqrt{3}$. The extra time needed for this is obviously undesirable but appears to be unavoidable if the catalog error turns out to be $\gtrsim 0\overset{m}{.}05$. Fortunately this procedure would not need to be continued indefinitely. After each star, on the average, had been observed three times corrections could be made to the catalog. The resulting catalog would have a strongly non-normal error distribution, but in the application for GEODSS could be considered to have its mean error reduced to $\lesssim 0\overset{m}{.}04$.

III.   ERROR

There are two basic sorts of error involved in GEODSS measurements of satellite brightness.   The first of these is inherent in the photon detection process and is generally called shot noise.   The second is due to the data manipulation involved in correcting for atmospheric extinction and reducing the data to a standard system, and will be referred to as system error. Shot noise and system error have very different properties and must be dealt with in different ways.

If an average of n_o photo-counts is observed in a series of one second measurements, the individual measurements will follow a Poisson distribution about n_o, with a standard deviation of $\sqrt{n_o}$.   The mean relative error of a single measurement will thus be

$$\varepsilon = \sqrt{n_o}/n_o = 1/\sqrt{n_o}$$

For a measurement of t seconds, the mean will be $n_o t$ and the standard deviation will be $\sqrt{n_o t}$ giving

$$\varepsilon = 1/\sqrt{n_o t}$$

This error, the shot noise, may thus be reduced by making a

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longer measurement. In the case of the usual astronomical type measurement, the object, with a mean count rate s, is observed against a background, with a mean count rate b, which is then separately measured and subtracted. We thus have

$$st = (st + bt) - bt$$

$$\epsilon = \sqrt{st + 2bt}/st$$

$$= \sqrt{1 + 2b/s}/\sqrt{st}.$$

The relative error in the measurement of an object is thus influenced by the brightness of the object, the brightness of the sky background, and the duration of the measurement. The most effective strategy for reducing the error is usually reduction of the background contribution by means of a field stop. Use of a field stop smaller than about 20 arc-seconds is generally diffi-cult and below 10 arc-seconds impossible, because of error in-troduced by poor centering due to tracking errors and atmospheric refraction.

System error may be further divided into error due to inac-curacies in the parameters used in the data reduction, "measure-ment error", and to inadequacies in the reduction technique, "modeling error".

GEODSS measurements will be corrected for atmospheric extinction and reduced to the standard system in a single step. Two measurements are made: the program object (satellite) and catalog standard (star). The measured magnitude, m', is related to magnitude in the standard system, m_o, by

$$m_{po} = m_p' + z - X_p k \qquad \text{for the satellite}$$

$$m_{co} = m_c' + z - X_c k \qquad \text{for the star}$$

where z is the zero-point drift, X is the air mass, and k is the extinction coefficient. The program object thus has

$$m_{po} = m_p' + (1 - \frac{X_p}{X_c})z - \frac{X_p}{X_c} m_{co} + \frac{X_p}{X_c} m_c'$$

$$\sigma_p^2 = \sigma_1^2 + (1 - \frac{X_p}{X_c})^2 \sigma_z^2 + (\frac{X_p}{X_c})^2 \sigma_1^2 + (\frac{X_p}{X_c})^2 \sigma_2^2$$

Where σ_1^2 is the variance in the measurements of a single object and σ_2^2 is the variance in the catalog of standard magnitudes. Now we can arrange for $X_p = X_c$ and z should be easily controllable to within a few percent, so we have:

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$$\sigma_p{}^2 = 2\sigma_1{}^2 + \sigma_2{}^2 \ .$$

There are several contributions to σ_1, amongst them atmospheric scintillation, decentration, and short term fluctuations in the preamplifier gain. These can all be kept quite small, and we shall use a value of $0\overset{m}{.}02$ for σ_1 . This small value does not come easily. One particular problem is the centering of the image in the field stop, especially for the faster satellites. In order to keep σ_1 small, the absolute tracking error, including the effects of atmospheric dispersion, should be better than one-fourth the field stop diameter. The catalog error, σ_2, is a bit more difficult to evaluate as it depends on the quality and homogenity of photometry of a large number of observers. This was discussed in Section II; for now we take $0\overset{m}{.}04$ as the standard deviation in the catalog magnitude. The total measurement error is thus $0\overset{m}{.}049$.

The technique used above incorporated a very simple model of the atmospheric extinction. In fact extinction is considerably more complicated and therefore the use of such a simple model introduces error. We will consider two effects: the non-linearity of extinction and the dependence of the extinction coefficient on spectral distribution.

Consider the case of a quadratic extinction law:

$$E_p = k'X_p + k''X_p^2 \ ,$$

$$E_c = k'X_c + k''X_c^2 \ .$$

The data reduction uses a linear estimate of the extinction,

$$\tilde{E}_p = (X_p/X_c)E_c \ ,$$

thus leading to an error in the extinction,

$$\delta E \equiv E_p - \tilde{E}_p$$

$$= k''X_p(X_p - X_c) \ .$$

This error is evaluated below for values of the second order coefficient appropriate to the GEODSS response profile.

Even within the linear approximation to the extinction, errors may arise because the extinction coefficient is not a universal constant but varies with the spectral distribution of the source being measured. Thus

$$E_p = k_p X_p$$

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$$E_c = k_c X_c.$$

Application of the linear estimate, $\tilde{E}_p$, defined above and having no color dependence, leads to an error

$$\delta E = X_p (k_p - k_c) \ .$$

In order to calculate a mean value for δE, it is necessary to have detailed knowledge of the reflectivities of the objects measured. This knowledge is not available; an estimate of δE was obtained in the following way. The atmospheric extinction may be calculated for a given source by integrating the product of flux, atmospheric transmission, and detector response with respect to wavelength. This integration was carried out for two source distributions, sunlight and sunlight reflected from gold, giving a value for the extinction error for the specific case of a gold colored satellite corrected for extinction via measurement of a solar type star. This value was then taken to be one-half the extreme range of a uniform distribution of δE's, leading to $\sigma_E = \delta E / \sqrt{3}$. While this is certainly plausible, it is only a guess as to the reflectivity properties of the satellite population. The value of $\tilde{\sigma}_E$ was calculated for two response profiles and used below to evaluate the extinction error due to the spectral distribution effect.

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In addition to the two inadequacies discussed above, at
least two more model failures will degrade the data. The model
implicitly assumes that extinction is not a function of any
geometric variable other than zenith distance (or elevation), nor
is it a function of time. In fact neither of these two assump-
tions is true except on "photometric" nights. Astronomers get
around this problem by simply declining to do photometry on any
except photometric nights, a strategy not available to GEODSS.
Proper site location can decrease the importance of these effects;
beyond that there is nothing that can be done to improve the
situation. It must, therefore, be borne in mind that even pessi-
mistic evaluations of data quality are optimistic in ignoring
this problem.

In order to evaluate the model errors discussed above, a
value for the airmass must be chosen. In doing this, we should
remember that the point in setting specifications is not to
provide parameters for error analysis, but to guarantee data
quality sufficient for the missions of GEODSS. Although a majority
of the ETS observations have been made above 30° elevation (X =
2), a significant fraction of the interesting objects are ob-
served between 30° and 20^{0} elevation (X = 3), with some observa-
tions carried out even lower (X as high as 5 or 6). Based upon
this a value of X = 2.5 has been chosen as the "baseline" airmass
for evaluation of these errors.

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Evaluation of the errors discussed above has been carried out for a photometric reference object which is defined in Appendix B. Calculations were carried out both for the reference object ("faint") and for an object $1^{m}.5$ brighter ("bright"). In addition, two response profiles were used: the "bare" gallium arsenide response and gallium arsenide plus a GG435 filter. The results are summarized below. The shot noise calculation was based on a five second measurement of background and a five second measurement of background plus object. Several features of the table should be noted. For the faint object shot noise is the most important source of error, while for the bright object system error dominates. The introduction of the GG435 filter produces a dramatic reduction in the error due to color effects. The filter also worsens the shot noise error slightly, but this is more than compensated for by the color effect reduction. The overall 2σ values are significantly improved for both objects by the inclusion of the GG435 filter.

TABLE I

ERRORS IN GEODSS PHOTOMETRY

ERROR SOURCE	FAINT			BRIGHT	
	BARE	GG435		BARE	GG435
Shot Noise	$0.^{m}078$	$0.^{m}085$		$0.^{m}020$	$0.^{m}022$
Measurement	.049	.049		.049	.049
Nonlinearity	.009	.003		.009	.003
Color Terms	.071	.027		.071	.027
RSS	.117	.102		.089	.060
2σ	.23	.20		.18	.12

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IV.   DETECTION OF SPIN PERIOD

One of the most important functions of GEODSS photometry will be the detection and measurement of spin or tumble periods for satellites. In addition to allowing assignment of motion class, the existence of periodicities can be extremely useful in confirming identities in searches for new objects. Periodicities may evidence themselves in either the specular or the diffuse signature. These cases have distinctly different properties and will be investigated separately. Since, however, neither case requires reduction to a standard system, only the shot noise need be considered.

The features of the specular signature are very short, typically a few milliseconds duration. In the analysis given below it shall be assumed that the signature may be divided into time segments (bins) of length equal to the duration of the specular feature with all the energy of the feature arriving in one bin. In order to reasonably approximate this situation in practice, the sampling time must be short compared to the bin size. A time of one millisecond per sample (i.e., a rate of 1kHz) should be adequate in most cases.

It turns out to be rather difficult to derive a specifica-
tion directly from the requirement to detect specular flashes.*
The false alarm and miss rates may, however, easily be evaluated
for a measurement with specific properties. For the photometric
measurement described in Appendix B, with the GG435 filter in
place, the signature would be a string of "empty" (i.e., con-
taining only background counts) bins, with a mean content
of $\mu_O = 46.0$, and occasional "full" (i.e., background plus specu-
lar flash counts) bins, with a mean content of $\mu_1 = 87.1$. The
simplest period detector would be a threshold detector which
transforms the signature to a string of 0 values with occasional
1 values and then reports the separation between the 1's. The
errors of this detector are empty bins erroneously reported as

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*McNamara and Seay (1) have developed inequalities describing the
errors of a threshold detector operating on digital data con-
taining Poisson noise. These are

$$\text{PFA} < \frac{1}{\sqrt{2\pi N}} \frac{N + 1}{N + 1 - \mu_O} (\frac{\mu_O}{N})^N e^{N - \mu_O}$$

$$\text{PM} < \frac{1}{\sqrt{2\pi (N - 1)}} \frac{[1 - (N - 1)/\mu_1]^N}{1 - (N - 1)/\mu_1} e^{N - 1 - \mu_1} (\frac{\mu_1}{N - 1})^{N - 1}$$

The direct way of proceeding would be to set acceptable false
alarm and miss rates, and eliminate the threshold, N, between the
equations leaving a required mean flash brightness, $\mu_1 - \mu_O$,
as a function of the background, μ_O. This cannot be done, so a
different approach must be found. Here the error rates have been
evaluated for a specific marginal detection to demonstrate that
this particular task is so easy that it is almost surely implicit
in the other system requirements.

1's (false alarms) and full bins erroneously reported as 0's
(misses). Using a threshold value of N = 73 yields a predicted
miss rate of about 7% and a predicted false alarm rate of about
1.5×10^{-4}. These rates seem quite satisfactory. In practice
specular flashes will usually be brighter and data processing
more sophisticated so that period detection by means of specular
flashes is probably the easiest of the photometric tasks.

At this point a comment about storage and transmission of
SOI signatures is appropriate. The shortest time of interest for
a satellite signature is the duration, τ, of the specular flash
from a flat surface rotating with the satellite, having spin
period T. These are related by $\tau = (3/4)T$, where τ is
in milliseconds and T is in seconds. It then follows that at
most $\frac{4}{3} \times 10^3$ data points per period are needed to describe the
signature. If the range between the threshold object to the
maximum counting rate of the equipment ($\sim$ 100 MHz) is compared to
the required accuracy of data, it is seen that each datum can be
represented in 1 byte (8 bits). The characteristic signature of
an object can thus be stored to the appropriate accuracy in
$\leq$ 2K bytes. That record could contain the signature presented as
"raw" data or as magnitudes, and all relevant timing and con-
ditions of observation information. The most difficult part of
this processing is a period detector. The software needed for
this is already required by the specification. The only addition

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necessary would be the rather straightforward synchronous integration program.

The feature of a diffuse signature is an approximately sinusoidally varying flux. Thus the target provides a mean photon flux, in alternate halfcycles, of $N = N_o(1 \pm f)$ where f is the fractional modulation. To this must be added the constant background flux, N_b. The period detection thus reduces to the detection of the difference between alternating measurements $(N_o(1 + f) + N_b)\tau$, and $(N_o(1 - f) + N_b)\tau$, where τ is the half-period. The difference is to be compared to the result for the case of no modulation ($f = 0$). The latter difference is, of course, zero in the mean, with variance $2(N_o + N_b)\tau$. The criterion for detection thus becomes

$$2fN_o\tau \geq \alpha\sqrt{2(N_o + N_b)\tau}$$

where α is a parameter adjusted to give a desired level of confidence for a given length of data. For example, setting $\alpha = 3$ should provide "sure" detection by eye in a single cycle. This expression can be combined with the expression defining shot noise. The resulting expression for shot noise in a measurement of length τ of the modulated signal provides an alternative way of specifying the sensitivity.

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$$\epsilon^2 = (N_o + 2N_b)/N_o^2 \tau$$

$$= 2(N_o + N_b)/N_o^2 \tau - 1/N_o \tau$$

$$= \frac{4f^2}{\alpha^2} - \frac{1}{2N_b \tau}\left[\sqrt{1 + 8\frac{f^2}{\alpha^2}N_b \tau} - 1\right]$$

$$= \left(\frac{2f}{\alpha}\right)^2 \left\{1 - \frac{\alpha^2}{8f^2 N_b \tau}\left[\sqrt{\frac{8f^2 N_b \tau}{\alpha^2} + 1} - 1\right]\right\}$$

$$\epsilon = \frac{2f}{\alpha} Q(N_b)$$

where $Q(N_b) \equiv \left\{1 - \frac{\alpha^2}{8f^2 N_b \tau}\left[\sqrt{\frac{8f^2 N_b \tau}{\alpha^2} + 1} - 1\right]\right\}^{\frac{1}{2}}$

is a very slowly varying function of N_b, essentially equal to unity for the range of interest. Reasonable values for a modulation just detectable via computer data processing are $f = .1$, $\tau = 1$, and $\alpha = 1$. The specification that such modulation be detectable is thus equivalent either to the requirement that

$$N_b \leq N_o^2/50 - N_o,$$

or the requirement that

$$\varepsilon \leq 0\overset{m}{.}20$$

for a 1s measurement of the object and a 1s measurement of background.

V.   MEASUREMENT OF PHASE FUNCTION

The stated aim of SOI photometry is the assignment of a more
or less detailed object classification.* There are three photo-
metric sources of information upon which to base such a classi-
fication: the signature of a spinning satellite, the spectral
distribution of the reflected sunlight, and the phase function.
Classification by signature has been studied and a few techniques
developed which show some promise. Examples of optical signa-
tures can be found in (7). Unfortunately, only a minority of
objects show a well defined signature. Study of the spectral
distribution also shows promise. In a report to be published the
application of a simple one dimensional classification based on
spectral distribution will be discussed. The remaining source of
information, the phase function, will be discussed here.

The brightness of an object is a complicated function of
object shape and observing geometry. The simplest set of assump-
tions (simple shape and perfectly diffuse reflection) lead to the
phase function (ψ)

$$\psi_0 = k \cos^2(\phi/2)$$

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*The more optimistic workers hope to see this develop into a
functional "picture" of the satellite. At the other end of the
range, even a simple binary classification such as familiar/un-
familiar should prove useful.

where ϕ is the sun-object-observer angle. This result is obtained for the maximum reflection from both a cylinder and a flat plate. A logical first try at assignment of object class would thus be based upon the measured departure from this phase function. In order to get a useful range of ϕ it will be necessary to combine observations taken over extended periods of time, at a variety of air masses, and possibly even from multiple sites. For these reasons it will be necessary to reduce all observations to a standard system; thus system error as well as shot noise must be considered.

W. E. Krag and W. J. Taylor (private communication) have made some measurements of the brightness of a variety of satellites. Their results can be used to estimate the quality of data needed to draw at least some conclusions as to object classification. The objects measured include several satellites which would be expected to have the simplest phase function and several which would be expected to have more complicated phase functions. We shall see if a simple discriminant can distinguish between the simple phase function and more complicated ones with the given data quality. Paragraph 3.7.1.4.2 of the specification states that the data quality has to be at least $0^m.25$ without relating this to a standard error. Here we shall consider the possibilities of considering this figure to be alternatively a 1σ or a 2σ value.

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Figure 1 shows Krag and Taylor's data plotted in three groups. The data were obtained under difficult conditions with a non-linear device. Two results of this are inhomogeneity in the data and excessive scatter in the fainter satellite measurements, those in the first group. Because of this the strictness of the test will have to be softened somewhat. Nonetheless, the conclusion reached seems reliable. The simplest discriminant is a direct comparison between the observations and the predicted values for the simple phase function. Using the 1σ interpretation of the $0^{m}.25$ figure from above, curves have been drawn in at $\pm 2\sigma$ from the simple phase function. In order to record an agreement with the simple phase function, 95% of the observations should lie between these limits. If significantly more than 5% of the measurements lie outside the limits, a disagreement would be flagged, and, of course, there is always the marginal region containing missed disagreements and incorrectly flagged agreements. The better the data, the smaller this marginal region.

The top group in Figure 1 is a composite of observations of SMS-2, SMS-3, WESTAR-1, and WESTAR-2. These objects are essentially simple cylinders and so would be expected to show the simple phase function to good approximation. In fact more than 5% of the points lie outside the limits. In this case, however, this seems easily attributable to the quality of the data
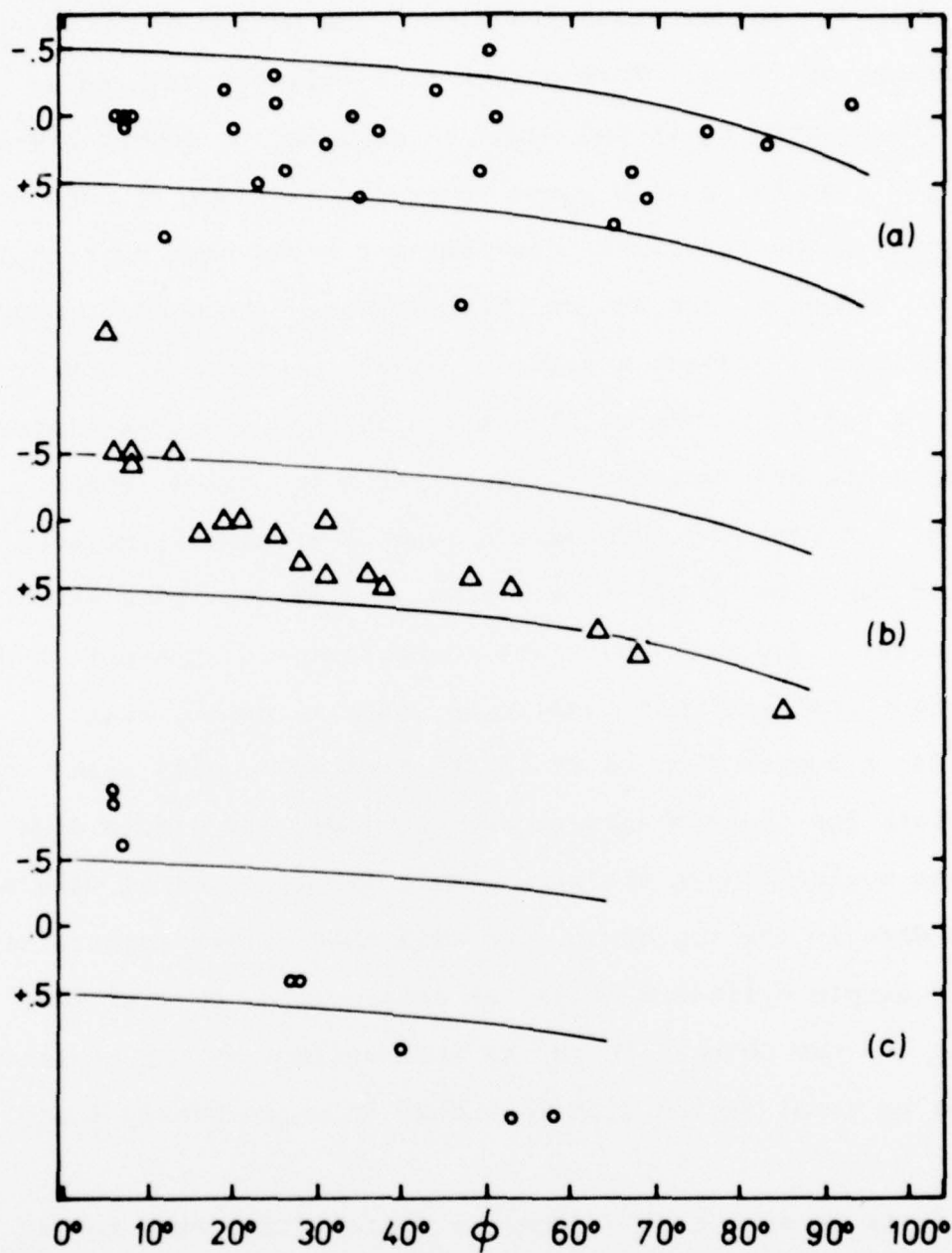
Fig. 1. Phase function for three groups of satellites compared to the simple Lambertian function $\psi = K \cos^2(\phi/2)$. The ordinate is astronomical magnitude brighter (−) or fainter (+) than the mean projected to $\phi = 0$ via the Lambertian function.

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rather than the actual phase function. The bottom group is observations of CTS-1. This object is a cylinder with large structures mounted on it and would be expected to depart significantly from the simple phase function. In fact it does so dramatically, and our simple discriminant would have no trouble correctly flagging this set of observations. The middle group of data in Figure 1 present a problem, however. These are observations of RCA-1 and RCA-2. These two objects also have large structures and are expected to depart from the simple phase function. In fact the observations show a strong resemblance to the phase function of CTS-1, but with smaller amplitude and a bit more scatter. The fact that this resemblance is apparent is due to the fact that the data quality is actually better than $1\sigma = 0\overset{m}{.}25$; a smooth pair of 2σ limit lines at $\pm \sim 0\overset{m}{.}25$ seems more appropriate for these observations. However, our simple discriminant would produce the same result for this set of data as for the data in the top group. We will thus either incorrectly flag the simple cylinders in (a) as interesting, or will miss flagging the two objects in (b) as interesting. We may conclude that the marginal region with $1\sigma = 0\overset{m}{.}25$ is unacceptably large.

This is an easy test - a barely useful first step toward assignment of object classification. If it cannot be easily performed there is little hope for this phase of GEODSS photometry. Interpretation of the $0\overset{m}{.}25$ figure from paragraph

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3.7.1.4.2 as a 1σ value is thus clearly ruled out. On the other hand, the limit curves suggested above for (b) at $2\sigma = 0\overset{m}{.}25$ provide a reasonable description for this phase function, and one which can be easily identified by our simple discriminant. It also seems fair to assume that if the quality of the data in (a) could be improved to $2\sigma = 0\overset{m}{.}25$, it would test as being consistent with the simple phase function. The overall conclusion, then, is that photometric data quality of $2\sigma = 0\overset{m}{.}25$ is both needed for and good enough to allow study of the phase function for the purpose of assigning object classification.

VI.   CONCLUSIONS

Paragraph 3.7.1.4.2 states the requirement that the data quality be good enough to allow accuracy of $\pm0^{m}\!.25$ for a threshold object. Paragraph 3.7.2.3.2 adds that the accuracy shall be $\pm0^{m}\!.125$ for an object $1^{m}\!.5$ brighter. All this is strictly equivalent to stating that the total error shall be composed of a brightness dependent part (shot noise) equal to $0^{m}\!.225$ at threshold and a brightness independent part (system error) equal to $0^{m}\!.112$. As was pointed out in section III, the error due to shot noise is a function of the duration of a measurement. The shot noise figure thus requires both a statement of relationship to the standard error and a timing parameter. The system error only requires the statement of relationship to the standard error.

In Section III, several sources of system error were detailed. Although under some conditions, some of the errors could be reduced in magnitude, it appears extremely unlikely that $0^{m}\!.112$ as a 3σ value for the GEODSS system error could be achieved under even the best of circumstances. Indeed, this figure would be more appropriate for photometry carried out in a program of astronomical research. In section V, however, we are led to demand that the $0^{m}\!.25$ figure be 2σ value. If the system error of $0^{m}\!.112$ is taken as a 1σ value it will be essentially impossible to achieve this. A 2σ interpretation thus seems to be forced upon

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us, and indeed we shall accept it for all comparison between the requirements developed here and the specification requirements. This isn't really so bad. Although $0^m.25$ viewed as 3σ is "never" exceeded, $0^m.25$ viewed as 2σ is exceeded only about 5% of the time.

Section V not only demands a certain precision of data but offers hope that data of that precision will be of some use in the determination of object classification. Until the actual assignment of object classification is on a firmer foundation, it is difficult to say any more about the kind of data needed. For those objects which are spinning or tumbling, the signature data needed to provide the period will also provide help in assignment of object classification. Finally, some preliminary studies indicate that multi-bandpass photometry will be useful in as-signment of object classification. The techniques for this are essentially non-existent at this time. Under the circumstances, nothing more favorable can be said about the achievement of the first-listed goal of GEODSS photometry.

The second goal, determination of motion class, is a natural by-product of the third, determination of spin/tumble period, and does not need a separate discussion. The discussion of section IV indicates that the measurement of spin period by means of a specular signature is so easily accomplished that we needn't

consider it further, provided only that sampling is required to be done at 1kHz. In the case of the diffuse signature, detection of a "reasonable" modulation of the reference object was shown to be equivalent to a $0^{m}.20$ limit on the shot noise.

The fourth goal, determination of maximum/average magnitude, is essentially trivial. The only apparent use for such magnitudes, other than those implicitly contained in the first three goals, is estimation of object size via

$$m = const + 5 \log r - 2.5 \log \rho A \psi.$$

From this equation we can see

$$\sigma_A^2 = A^2 (.4 \ln 10)^2 \sigma_m^2 + 4 \left(\frac{\sigma_r}{r}\right)^2 + \left(\frac{\sigma_\rho}{\rho}\right)^2 + \left(\frac{\sigma_\psi}{\psi}\right)^2$$

But $(\sigma_\rho/\rho) \sim (\sigma_\psi/\psi) \sim 1$, so σ_m will never be an appreciable source of uncertainty in estimates of A.

To summarize the requirements considered necessary for the attainment of the GEODSS photometric goals:

1) Error due to shot noise in a 1 second observation of the reference object $\leq 0^{m}.20$ (1σ).

2) System error involved in transforming to the standard photometric system $\leq 0^{m}.055$ (1σ). Components which depend upon

air mass are to be evaluated at an air mass of 2.5. A realistic range of spectral distributions must be considered in evaluating the color effect.

3) Provision made for insertion of one of four filters into the optical path of the photometer.

4) Sampling rate $\geq$ 1kHz.

For comparison, the current estimates of the two errors specified above for the case of the measurement described in the appendix are respectively $0\overset{m}{.}17$ and $0\overset{m}{.}087$ for wide open GaAs and $0\overset{m}{.}19$ and $0\overset{m}{.}056$ if the GG435 filter is added. On the basis of this and the error budget presented in section III, it appears that the requirements derived here are within the capablity of the hardware currently envisioned for *GEODSS*, consonant with a 2σ reading of paragraph 3.7.2.3.2, and satisfactory with respect to the goals of paragraph 3.1.2.4.

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APPENDIX A

EXCERPTS FROM THE SPECIFICATION

Quoted here are several paragraphs of reference (8) which deal directly with the question of photometric precision in the GEODSS system.

3.1.2.4 Space object identification (SOI). This mission is to collect, process, edit and forward radiometric data to the ADCOM Intelligence Center (ADIC) in NCOC, including the results of on-site elementary analysis of radiometric signatures to determine general characteristics, i.e., object classification (payload, tank, fragment, unknown), motion classification (stable, tumble, spin), period, maximum magnitude/average magnitude of the object being tracked.

3.7.1.4 Radiometer. A radiometric sensor shall be associated with each telescope. It shall provide a time history of the optical radiation from a satellite, and shall produce data from which target characteristics such as magnitude and spin rate may be deduced. The radiometer shall also be used for sampling the night sky background level, and for measuring atmospheric extinction. This instrument shall meet the following requirements.

3.7.1.4.1 Sensitivity. The radiometer shall meet the same sensitivity requirements, at a given level of night sky back-

ground, as the surveillance sensor specified in 10.3.1.

3.7.1.4.2 Accuracy. Data from the radiometer shall be of a quality such that specular and diffuse magnitudes of a target may be determined to an accuracy of $\pm 0^m.25$ by on-site processing.

3.7.2.3 Radiometric signal processing. This function shall provide the capability to record, display, and combine sensor data to determine the optical radiation signature of a satellite, the night sky background signal, and the atmospheric extinction.

3.7.2.3.2 Recording. The processed target return signal (3.7.2.3.1.4) shall be recorded digitally, along with time (within ± 1ms, blocked once per second) and instrument settings necessary for data reduction. SOI color indexing information (see 3.2.1.1.3.1) shall be recorded, and provisions shall be made for recording the filters used. The "star alarm" signal (3.7.1.4.6) shall also be recorded, as shall the night sky background signal (3.7.2.3.1.2). (Note that the target signature, as recorded here, includes the background component, which is recorded separately for later processing.)

3.7.2.3.4 Further on-site radiometric processing. The data described in 3.7.2.3.2 will be processed by ADIC, however, each site shall have the following limited capability for SOI: calculating the diffuse and specular magnitude, to an accuracy of $0^m.25$, at the threshold sensitivity and $0^m.125$ for a target brightness of $1^m.5$ greater than threshold sensitivity, and determining

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any basic periodicities exhibited by the space object. In addition, each site shall maintain a library of signatures for all those objects on which SOI data have been obtained.

APPENDIX B

PHOTOMETRIC REFERENCE MEASUREMENT

The photometric reference used here is a sphere with diameter 1 meter and diffuse reflectivity of 10% with an attached square flat plate with edge 1 cm and specular reflectivity of 15%. The object is assumed to be at synchronous distance and viewed at a phase angle of 20°. A rotation period of 4s is used, leading to a duration of specular flash of 3 ms. The object is viewed in a sky with a brightness of $19^m.5$/sq.arc-sec.

The reference object is viewed with a telescope/photometer having the following parameters: effective area = $.45m^2$, throughput to photometer = 80%, field stop for photometer = 20 arc-sec, and photocathode is gallium arsenide with a peak quantum efficiency of 30%. Assuming the spectral distribution of sunlight yields detected photon fluxes as measured through 1.5 air masses as follows:

    sphere: 1093 /s

    plate: 16500 /s

    sky: 17400 /s

For the case of the addition of the GG435 filter, these numbers should be multiplied by 0.83.

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Requirements on the accuracy of GEODSS photometry were derived based on the stated goals of GEODSS photometry. Fulfilling these requirements will ensure that the data are of sufficient quality for their intended purpose. Calculations based on the properties of the proposed system show that the requirements can be met if care is taken in the detailed design and data processing techniques.

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